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# SPATIAL FRACTIONATION OF PARAMAGNETIC PARTICLES PRECIPITATING IN A HIGH-GRADIENT MAGNETIC FIELD OF A SHORT MAGNETIZED CYLINDER

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*The motion of dia- and paramagnetic particles precipitating in a liquid near a vertical magnetized cylinder of finite length in a plane transverse to the direction of magnetization has been studied, the phenomenon of spatial separation of paramagnetic particles by the value of magnetic susceptibility is described, and a comparison of the efficiency of separation of paramagnetic particles in the fields of a short cylinder and a sphere has been carried out.*

***Keywords:*** *paramagnetic microparticles, magnetophoresis, high-gradient magnetic separation.*

**Introduction.** The difference in the magnetic properties of materials has long been employed for magnetic separation in many branches of industry, including the concentration of ores, separation of metal-working waste, water treatment, and purification of process liquids and of the products of chemical and pharmaceutical production [1]. Traditional applications are connected with large-scale productions and materials possessing strong magnetic properties. At the same time, the application of magnetic fields combining a high magnitude and strong small-scale inhomogeneity allows one to first of all separate even microscopic slightly magnetic objects, such as biological cells, from a liquid or gaseous medium [2–4]. Recently, in view of the rapid development of cell technologies in biology and medicine, investigation of magnetic separation of cells has become very topical [5]. Interesting prospects are connected with the use of magnetophoretic processes for the analysis of fine disperse materials in industry and biology. In particular, the possibility of carrying out magnetic microscopic control of uranium ore [6] has been reported, and the substantial dependence of the magnetic properties of erythrocytes on the degree of their saturation with oxygen [3] and the differences in the magnetic properties of different components of a suspension of insulin-producing cells of the pancreas of a rabbit [7] and the change in the magnetic properties of the cells of the spleens of mice in the process of development of a malignant tumor in them [8] have been revealed. To obtain a more profound knowledge of the possibilities of the magnetic methods of separation and metrology of slightly magnetic microparticles, it is necessary to comprehensively study the processes of magnetophoresis in various-purpose high-gradient magnetic fields. As a rule, such fields are produced near small ferromagnetic bodies magnetized by an external strong homogeneous field. In the literature (see [5]), mainly the problems of motion and capture of slightly magnetic particles by spherical or infinite cylindrical bodies have been considered. Investigation of the motion of particles precipitating in a liquid under the gravity force in the vicinity of a magnetized cylinder of finite length [9] has revealed interesting trends pointing to the possibility of using the given geometry for spatial separation of diamagnetic particles and analyzing their distribution by magnetic properties. In [9], the case of the motion of particles in a vertical plane formed by the direction of the field and the axis of the vertical cylinder is considered. The spatial fractionation of paramagnetic particles in such a situation is possible during their motion in the plane perpendicular to the external field. This problem is considered in the present work.

**Statement of the Problem and Calculation of the Magnetic Field.** The geometry of the problem is presented in Fig. 1. We consider the motion of a slightly magnetic particle in a viscous liquid caused by the action of gravitation and high-gradient magnetic fields. To produce a high-gradient field, a vertical ferromagnetic cylinder of length $2L$ and diameter $2a$ is magnetized to saturation by an external homogeneous magnetic field $\mathbf{H}_{ex}$ applied across the cylinder axis. We introduce a coordinate system with origin at the geometric center of the cylinder (Fig. 1) and

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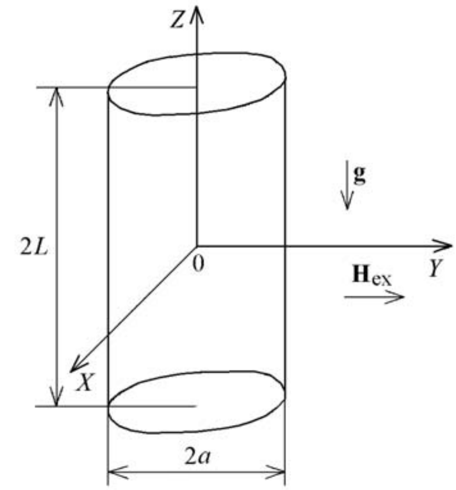



Fig. 1. Geometry of the problem.

consider the motion of particles in the plane $Y = 0$. The resultant field in the system can be represented as the sum of a magnetizing homogeneous field and the field $\mathbf{H}'$ produced by the magnetized cylinder.

Using the cylinder's radius as the scale of the distance, we determine the nondimensional coordinates $x = X/a$, $y = Y/a$, and $z = Z/a$. Performing calculations similar to those in [9], in view of the symmetry of the considered cylinder–external field system we obtain that the self-field of the cylinder has the sole nonzero component $H_y'$ in the plane $y = 0$. Scaling the field intensity by $2\pi I_s(\mathbf{h} = \mathbf{H}/2\pi I_s)$ at a certain point A, we have

$$h_y' = -\frac{1}{2\pi}\iiint_v \frac{1}{r_{AB}^3}\left[1 - 3\frac{y_B^2}{r_{AB}^2}\right] dx_B dy_B dz_B\,, \quad r_{AB}^2 = (x - x_B)^2 + y_B^2 + (z - z_B)^2\,. \tag{1}$$

The subscript B designates the point belonging to the cylinder. Evaluating the integral (1) from the variables $z_B$ and $y_B$ in an explicit form, we arrive at the equation

$$h_y' = \int_{-1}^{1} [f_1\,(x_B, x, z, l) + f_1\,(x_B, x, -z, l)]\, dx_B\,. \tag{2}$$

Here $l = L/a$;

$$f_1\,(x_B, x, z, l) = -\frac{1}{\pi}\left(\frac{1 - y_B^2}{1 + x^2 - 2xx_B + (l + z)^2}\right)^{1/2} \frac{l + z}{1 + x^2 - 2xx_B}\,.$$

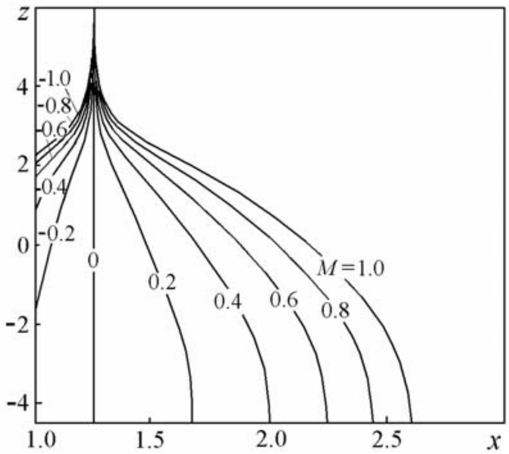
The motion of a particle in the inertialess approximation is described by the equation

$$\mathbf{F}_m - 3\alpha\pi d\eta\,\frac{d\mathbf{R}}{dt} + \mathbf{g}\Delta\rho V = 0\,, \quad \Delta\rho = \rho - \rho_0\,, \tag{3}$$

which represents the condition of mutual compensation of the magnetic, sedimentation, and viscous forces. Subject to the condition that the scale of the magnetic field inhomogeneity is large compared to the particle dimensions, the magnetic force is given by the relation [10]

$$\mathbf{F}_m = \frac{1}{2}\Delta\chi V\nabla\mathbf{H}^2\,, \quad \Delta\chi = \chi - \chi_0\,, \tag{4}$$

Following [11], we introduce the magnetophoretic potential of the field $\Phi$ according to



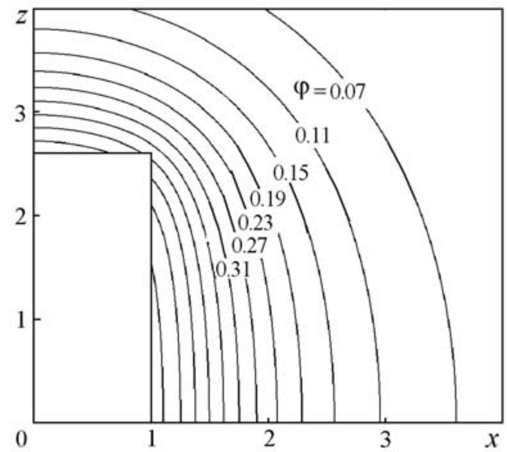




Fig. 2. Trajectories of particles with different values of $M$. The impact parameter $\delta = 1.25$, the cylinder half-length $l = 2.6$ of the radius, and the nondimensional strength of the field $P = 1.5$.

Fig. 3. Equipotential lines of the magnetophoretic cylinder $\varphi$ of a transversely magnetized cylinder in the plane $y = 0$ (cylinder half-length $l = 2.6$ of the radius, nondimensional strength of the field $P = 1.5$).

$$\mathbf{F}_\mathrm{m} = -\nabla\Phi\,, \quad \Phi = -\frac{1}{2}\Delta\chi V \mathbf{H}^2\,. \tag{5}$$

Having discarded the constant quantity $\sim\mathbf{H}_\mathrm{ex}^2$ in the potential, we write

$$\Phi = -\frac{1}{2}\Delta\chi V(\mathbf{H}'^2 + 2\mathbf{H}_\mathrm{ex}\mathbf{H}') = -\Phi^*\varphi\,, \quad \varphi = -h_y'^2 - Ph_y', \quad \Phi^* = 2\Delta\chi V(\pi I_\mathrm{s})^2\,, \quad P = \frac{|\mathbf{H}_\mathrm{ex}|}{\pi I_\mathrm{s}}\,. \tag{6}$$

If we apply $\Phi^*$ as the force scale, we will obtain the nondimensional magnetophoretic force:

$$\mathbf{f}_\mathrm{m} = \frac{\mathbf{F}_\mathrm{m}}{\Phi^*} = -\nabla\varphi\,. \tag{7}$$

We pass on to nondimensional coordinates and break down the equation of motion (3) in the plane $y = 0$ into the components

$$3\alpha\pi d a\eta\frac{dz}{dt} = -\Delta\rho g V - \frac{2\Delta\chi V(\pi I_\mathrm{s})^2}{a}\frac{\partial\varphi}{\partial z}\,, \quad 3\alpha\pi d a\eta\frac{dx}{dt} = -\frac{2\Delta\chi V(\pi I_\mathrm{s})^2}{a}\frac{\partial\varphi}{\partial x}\,. \tag{8}$$

Eliminating the time in Eq. (8), we arrive at the following equation of the trajectory of the particle in the plane $y = 0$:

$$\frac{dz}{dx} = \left(\frac{\partial\varphi}{\partial x}\right)^{-1}\left[\frac{1}{M} + \frac{\partial\varphi}{\partial z}\right], \quad M = \frac{2\Delta\chi\,(\pi I_\mathrm{s})^2}{a\Delta\rho g}\,. \tag{9}$$

**Analysis of the Motion of Particles.** A particle whose density is higher than the liquid density, in the absence of a magnetic field settles down along the vertical line located at a certain distance from the cylinder axis $\delta > 1$ (the impact parameter). The bending of the trajectories of particles having different values of the magnetophoretic parameter $M$ under the action of the magnetic field of the cylinder is illustrated in Fig. 2 for the case where $\delta = 1.25$, $l = 2.6$, and $P = 1.5$. As is seen, paramagnetic particles ($M > 0$) are repelled by the cylinder, whereas diamagnetic particles are attracted by it. Beginning from a certain value of $M < 0$ the capture of diamagnetic particles by the cylinder is observed.

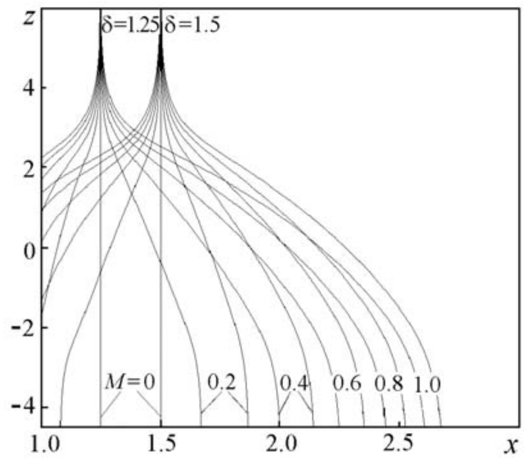



Fig. 4. Trajectories of particles obtained for different values of the magnetophoretic parameter $M$ and impact factors $\delta = 1.25$ and 1.5 (cylinder half-length $l = 2.6$ of the radius, nondimensional strength of the field $P = 1.5$).

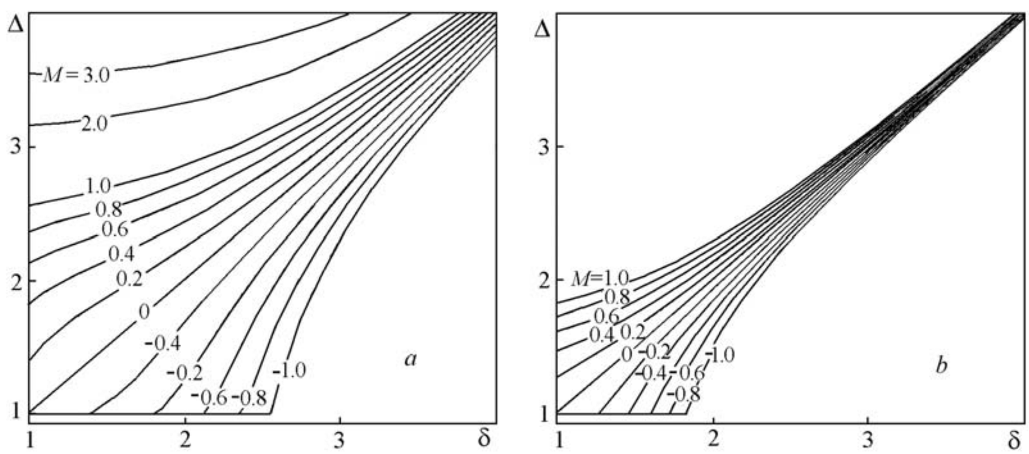



Fig. 5. Dependence of the exit distance on the impact parameter at different values of the magnetophoretic parameter $M$: a) for a cylinder; b) for a sphere.

The described behavior of particles is attributable to the character of change of the magnetophoretic potential, the distribution of which for the above-considered example is presented in Fig. 3 by equipotential lines. The paramagnetic particles experience the action of the magnetic force directed to the side of the decreasing potential, or, as is the case, to the side of diverging equipotential lines, whereas the diamagnetic line — to the side of the crowding lines.

Figure 4 presents two families of trajectories obtained for identical sets of values of the magnetophoretic parameter $M$ and different values of the impact parameter ($\delta = 1.25$ and 1.5). For magnetic separation, of interest is the influence of the cylinder on the motion of paramagnetic particles, viz.: the particles with the same values of the magnetophoretic parameter get closer together when passing near the cylinder, with the difference between the coordinates of the exit decreasing with increase in the absolute value of $M$. The effect of the convergence of paramagnetic particles with identical values of $M$ at the exit can be described by the dependence

$$\Delta = f(\delta, M) . \tag{10}$$

The family of the $\Delta(\delta)$ curves obtained for a number of values of $M$ are presented in Fig. 5a. The horizontal straight line $\Delta = 1$ in this figure corresponds to the deposition of particles on the cylinder surface. The separation of particles is the more efficient, the weaker the dependence of the exit distance on the impact parameter, i.e., the flatter the $\Delta(\delta)$ curve. As is seen from Fig. 5a, the efficient spatial separation of paramagnetic particles by magnetic properties is pos-

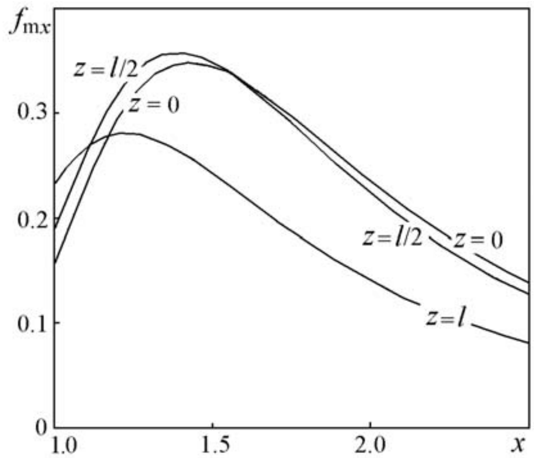



Fig. 6. Change in the horizontal component of the nondimensional magnetophoretic force.

TABLE 1. Values of the Magnetophoretic Parameter $M$ for Some Substances

| Substance | Magnetic susceptibility $\chi$, $10^{-6}$ | Density $\rho$, $g \cdot cm^{-3}$ | Carrying medium in which a particle moves | Parameter $M$ |
|---|---|---|---|---|
| Water | –0.72 | 1.00 | — | — |
| Aluminum | 1.64 | 2.69 | Water | 1.62 |
| Molybdenum | 9.50 | 10.22 | » | 1.28 |
| Zirconium | 8.90 | 6.45 | » | 2.05 |
| Niobium | 20.05 | 8.57 | » | 3.19 |
| Titanium | 14.36 | 4.50 | » | 5.01 |
| Uranium oxide (IV), crystalline | 47.08 [6] | 10.95 | » | 5.59 |
| Uranium-$\alpha$ | 33.36 | 19.4 | » | 2.16 |
| Medium No. 199 | –0.60 [11] | 1.04 [11] | — | — |
| Hela cells | –0.515 [11] | 1.25 [11] | Medium No. 199 | 0.47 |
| Erythrocytes containing deoxyhemoglobin | –0.526 [12] | 1.09 [13] | Medium No. 199 | 1.72 |

sible only on condition of rather high values of the magnetophoretic parameter $M$ ($M \geq 1$). At small values of $M$ and small values of the impact parameter, the influence of the cylinder can have the opposite effect — the particles having identical $M$ diverge. In Fig. 5a this is seen as distortion of the $\Delta(\delta)$ curves for $\delta \rightarrow 1$. The effect is explained by the specific change in the horizontal component of the magnetophoretic force $f_{mx}$ near the cylinder surface (see Fig. 6).

Additionally, calculations of the dependence of the exit distance on the impact parameter were carried out for the field of a magnetized sphere (Fig. 5b). For this field the size of the region where the force acting on precipitating particles is appreciable is smaller than in the cylinder. In the cylinder this region is extended along its axis, due to which, in contrast to the sphere, the particles, while moving vertically, do not leave this region and are longer subjected to the action of the magnetophoretic force, resulting in their greater horizontal displacement. This points to the fact that in practice it is better to use cylindrical ferromagnetic particles for producing a high-gradient field. In comparing Fig. 5a and b, the above-described differences manifest themselves in the curves of the exit distance $\Delta(\delta)$ for the sphere being located more compactly than those for the cylinder.

**Conclusions.** The results of numerical simulation of the motion of slightly magnetic microparticles near a short magnetized cylinder in the plane transverse to the direction of magnetization point to the possibility of using this geometry for separating paramagnetic particles into fractions and analyzing their distribution by the magnetic properties. Efficient fractionation of paramagnetic particles occurs at $M \geq 1$. The values of the parameter $M$ for some materials with well-known properties in the case of a cylinder of diameter 1 mm and intensity of magnetization 1700 Gs are presented in Table 1. These data point to the vast possibilities of applying the considered scheme in practical separation of paramagnetic particles.

## NOTATION

$a$, radius of the cylinder, cm; $d$, diameter of a particle, cm; $\mathbf{F}_m$, magnetic force, dyn; $\mathbf{f}_m$, magnetic force, nondimensional value; $f_{mx}$, $x$ component of the magnetic force vector; $\mathbf{g}$, vector of free-fall acceleration, cm·s$^{-2}$; $g$, free-fall acceleration, cm·s$^{-2}$; $\mathbf{H}$, vector of the magnetic field strength, Oe; $\mathbf{H}_{ex}$, vector of the strength of external homogeneous magnetic field, Oe; $\mathbf{H}'$, vector of the magnetic self-field strength of a magnetized cylinder, Oe; $H_y'$, $y$ component of the vector of the magnetic self-field of a magnetized cylinder, Oe; $\mathbf{h}$, vector of the magnetic field strength; $h_y'$, $y$ component of the vector of the self-field strength of a magnetic cylinder; $I_s$, saturation magnetization of the cylinder, Gs; $L$, half-length of the cylinder, cm; $l$, half-length of the cylinder; $M$, magnetophoretic parameter; $P$, external field strength; $\mathbf{R}$, radius vector, cm; $r_{AB}$, distance between points A and B; $t$, time, s; $V$, volume of a particle, cm$^3$; $v$, region of integration geometrically corresponding to a cylinder; $X$, $Y$, $Z$, Cartesian coordinates, cm; $x$, $y$, $z$, Cartesian coordinates; $\alpha$, hydrodynamic parameter of the shape of a particles; $\Delta$, exit distance; $\delta$, impact parameter; $\Delta\rho$, difference of densities of a particle and liquid, g·cm$^{-3}$; $\Delta\chi$, difference of magnetic susceptibility of a particle and liquid; $\eta$, viscosity of liquid, Pz; $\rho$, density of a particle, g·cm$^{-3}$; $\rho_0$, density of liquid, g·cm$^{-3}$; $\Phi$, magnetophoretic potential, Erg; $\Phi^*$, scale of magnetophorestic potential, Erg; $\varphi$, magnetophoretic potential; $\chi$, magnetic susceptibility of a particle; $\chi_0$, magnetic susceptibility of liquid. Subscripts: 0, liquid; ex, external; m, magnetophoretic; s, saturation; $x$, $y$, $z$, components of a quantity along the respective coordinate.

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